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During a Blow-out in an H₂ or Air Storage Cavern**

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A SIMPLIFIED SOLUTION FOR GAS FLOW DURING A BLOW-OUT IN AN H₂ OR AIR STORAGE CAVERN

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Abstract

A small number of blow-outs from gas storage caverns (for example, in Moss Bluff, Texas and Fort Saskatchewan, Canada) have been described in the literature. Gas flow lasted several days before the caverns were empty. In this paper, we suggest simplified methods that allow for computing blow-out duration and evolution of gas temperature and pressure in the cavern and in the well. This method is used to compute air flow from a shaft mine, an accident described by Van Sambeek (2009). The case of a hydrogen storage cavern also is considered, as it is known that hydrogen depressurization can lead, in certain cases, to hydrogen temperature increase.

Key words: blow-out, salt caverns, computer modeling, air, H₂, thermodynamics.

1. Introduction

The Moss Bluff blow-out is studied in the framework of an SMRI RFP; numerical computations using the actual state equation of natural gas are performed. In the present paper, simplified state equations (ideal gas and van der Waals gas) are used because they allow getting closed-form solutions of the blow-out problem. These solutions are applied to the cases of compressed air storage and hydrogen storage in salt caverns.

Compressed Air Energy Storage (CAES) is experiencing a rise in interest, as it can be used as buffer energy storage in support of intermittent sources of renewable energy, such as wind mills. CAES facilities are designed to deliver full-power capacity in a very short time period. There are two existing CAES in the world, in Germany (Huntorf) and in the U.S. (McIntosh). The Huntorf CAES facility is operated between 7 and 4.3 MPa, or 1000 and 600 psi (Crotogino et al., 2001). Research is active in France within the frame of the SACRE Program supported by ANR (Agence Nationale de la Recherche) and in Germany within the frame of the ADELE project (Bannach and Klafki, 2012).

Salt caverns storing hydrogen (ambient temperature, gas pressure in the 7 – 21 MPa or 1015 – 3046 psi range) are operated in the UK (Teesside, three 70,000-m³ or 0.45 MMbbls caverns at a 370-m or 1215 ft depth) and in Texas in the U.S. (Clemens Dome, Mont Belvieu, Moss Bluff). Hydrogen storage raises interesting problems, as its state equation and thermo-dynamical potentials differ significantly from those of an ideal gas.

2. GAS PRESSURE AND TEMPERATURE EVOLUTION DURING A BLOW-OUT

2.1. Coupling between the cavern and the well

In the case of a gas-exploration or production well, a blow-out may last several weeks or months, as the amount of gas in the gas reservoir is huge, making pressure decrease a very slow process. In gas-storage caverns, the gas inventory is much smaller, and cavern pressure drops to atmospheric pressure within a few days. As a consequence, thermo-dynamical behaviors of the gas in the cavern and of the gas in a borehole are strongly coupled; they will be discussed in Sections 2.2 and 2.3.

2.2. Flow of gas in the borehole

2.2.1. Main assumptions

It is assumed that there is no gas leak to the rock mass during the blow-out.

Steady-state flow

Gas rate in the borehole typically is a couple hundreds of meters per second (more, when hydrogen is considered). In other words, only a few seconds are needed for gas to travel from cavern top to ground level. Such a short period of time is insufficient for cavern pressure and temperature change significantly. Steady-state flow is assumed.

Adiabatic flow

Gas temperature decreases in the cavern and in the borehole, with possible serious consequences for the well. (Casing steel, cement and rock at the vicinity of the well experience large temperature changes, thermal contraction and development of tensile stresses.) Conversely, the amount of heat transferred from the rock mass to the gas is not able to change gas temperature significantly, as gas flow rate is extremely fast. Heat transfer from the rock mass is neglected, and gas flow is considered adiabatic.

Turbulent flow

At least during the first hours or days after the blow-out begins, gas flow is turbulent. The effects of friction are confined to a thin boundary layer at the steel casing wall. The average gas velocity is uniform through any cross-sectional area (except, of course, in this boundary layer). When z is the vertical coordinate ($z = 0$ at cavern top and $z = H$ at ground level) and short periods of time (a few minutes) are considered, the average gas velocity is a function of z (and not of t , as steady-state flow is assumed), or $u = u(z)$.

Duct

Duct diameter, or D , is assumed to be constant all along the well; hence, the cross-sectional area of the duct, S , is constant too.

These assumptions are commonly accepted (Ma et al., 2011); they are part of the so-called “Fanno-flow” model.

2.2.2. Equations

Gas flow can be described by the following set of equations:

$$P = P(v, T) \tag{1}$$

$$h = h(T, P) \quad (2)$$

$$u/v = u_0/v_0 = \dot{m} = \dot{m}/S \quad (3)$$

$$\frac{dh}{dz} + u \frac{du}{dz} + g = 0 \quad (4)$$

$$v \frac{dP}{dz} + u \frac{du}{dz} + g = -f(u) \quad (5)$$

$$u \frac{dS}{dz} \geq 0 \quad (6)$$

Equation (1) is the state equation that defines gas pressure (P) as a function of gas specific volume ($v = 1/\rho$, ρ is the gas density) and of gas (absolute) temperature (T).

Equation (2) defines the gas enthalpy per unit of mass (h) as a function of pressure and temperature.

Equation (3) is the mass conservation equation; u_0 and v_0 are gas celerity and specific volume at the casing shoe (or $z = 0$), respectively; m is the mass of gas in the cavern.

Equation (4) is the energy equation; g is the gravity acceleration.

Equation (5) is the momentum equation. Head losses per unit of length are described by $f(u) > 0$; $f = f(u; D, \varepsilon \dots)$ is a function of gas velocity, duct diameter, wall roughness, etc.

Statement (6) is the condition of positivity of entropy (S) change, which plays an important role in the context of the Fanno-flow model.

2.2.3. Boundary conditions and subsonic flow

Pressure and temperature in the cavern ($z = 0$), or P_0 and T_0 , are assumed known at any instant. Then, gas specific volume, v_0 , can be computed through the state equation $P_0 = P(v_0, T_0)$. Equations (1) to (5) allow computation of gas pressure and gas temperature in the borehole. In principle, gas pressure at ground level, or P_H , should be atmospheric: $P_H = P_{atm}$. However, this boundary condition cannot always be satisfied. From thermodynamics textbooks, it is known that: $dh(S, P) = TdS + v dP$; hence, $dh(S, v) = TdS - c^2 dv/v$, where c is the celerity of sound. (For an ideal gas, $c^2 = \gamma P v$). In a Fanno flow, when gravity is disregarded, $dh + u du = 0$ and $TdS = (c^2 - u^2) dv/v$. However, the sign of dS must not change; in other words, when the flow of gas is subsonic ($u_0 < c_0$) in the cavern, it must remain subsonic ($u < c$) in the borehole (except perhaps at ground level, or $z = 0$).

When applying the boundary condition, $P_H = P_{atm}$ leads to a solution such that gas flow is supersonic in a part of the well (This may occur when cavern pressure is high enough.), another solution must be

selected. This solution is constructed such that $u_H = c_H$. (The flow, which is sonic at ground level, is said to be “choked”.) In such a case, no constraint must be applied to P_H , which, in general, is larger than atmospheric. Conversely, when gas cavern pressure is relatively small, the flow is said to be normal: even at ground level, the gas rate is significantly slower than sound celerity and the boundary condition $P_H = P_{atm}$ applies.

2.2.4. Simplifications

The following simplified version of this set of equations allows obtaining a closed-form solution (discussed in the Appendix).

1. Body forces are disregarded, $g = 0$.
2. Head losses can be written: $f(u) = Fu^2$, where $F = f/2D$ is the friction coefficient, D is the diameter of the well (in meters or inches), and f is the friction factor.
3. Gas state equation $P = P(v, T)$ is simplified as follows. Two cases are considered. In the first case, the gas (typically, air) is ideal — i.e., its state equation is $Pv = rT$, $r = C_p - C_v$, $\gamma = C_p / C_v$, and its enthalpy can be written $h = C_p T$, where the gas heat capacity at constant pressure C_p is constant. In the second case, the gas (typically, hydrogen) is of the van der Waals type — i.e., its state equation is $P = -a/v^2 + RT/(v - b)$, where a , b are two constants, and its enthalpy is $h = C_v T - 2a/v + rT v/(v - b)$, where gas heat capacity at constant volume C_v is constant.

The properties of gases used in this paper are presented in Table 1.

Table 1: Characteristics of gases used in computation.

| Gases | C_p (J/kg/K) | C_v (J/kg/K) | γ (-) | M (g/mol) | a (J.m ³ /kg ²)* | b (m ³ /kg)* |
|----------|----------------|----------------|--------------|-------------|---|---------------------------|
| Air | 1010 | 719 | 1.40 | 28.95 | – | – |
| Hydrogen | 14,831 | 10,714 | - | 2.016 | 6,092 | 0.013 |

(*) a and b are the two coefficients of the van der Waals state equation.

2.2.5. Model assessment

The aim of this paper is to provide a clear picture of the main phenomena that affect gas flow during a blow-out, although results are indicative rather than exact. In fact, the model suffers from two flaws: (1) head losses are roughly estimated and, in fact, the actual coefficient F is a function of the flow rate, u ; and (2) even the van der Waals state equation is a less than perfect description of the actual behavior of hydrogen, and heat capacity, or C_v , is a function of temperature. Better results can be obtained through numerical computations.

2.3. Evolution of gas temperature and pressure in the cavern

Gas temperature, $T_0(t)$, and gas pressure, $P_0(t)$, can be considered as almost uniform throughout the entire volume of a cavern (Bérest et al., 2012). Gas kinetic energy in the cavern is neglected. During gas withdrawal, the heat balance equation can be written (ATG, 1986)¹:

$$m(\dot{e} + P\dot{v}) = Q \quad (7)$$

where m is the mass of gas in the cavern, and e is the gas internal energy. During gas withdrawal, cavern volume, or V_0 , only experiences a small change and can be considered constant, or $V_0 = m\dot{v}$ and $m\dot{v} = -\dot{m}v$. Q is the heat flux transferred from the rock mass to the cavern gas through the cavern wall, or

$$Q = \int K \frac{\partial T}{\partial n} da \quad (8)$$

where K is the thermal conductivity of salt. The heat flux from the cavern can be computed as follows.

The evolution of temperature in the rock mass is governed by thermal conduction. Generally speaking, penetration of temperature changes in the rock mass is slow. For instance, when a cold gas temperature T_0 has been kept constant during a t -long period of time on the (flat) surface of a half-space whose initial temperature (at $t = 0$) was T_∞ , temperature evolution can be written $T(x,t) = T_\infty + (T_0 - T_\infty) \operatorname{erfc}(x/2\sqrt{kt})$, where $k \approx 3 \times 10^{-6} \text{ m}^2/\text{s}$ is the rock thermal diffusivity. Heat flux at cavern wall is $Q = K(T_\infty - T_0)/\sqrt{\pi kt}$. Rock temperature changes significantly [by more than $\operatorname{erfc}(1/2) = 50\%$] in a domain at a cavern wall with a thickness of $d = \sqrt{kt}$, or $d \approx 1 \text{ m}$ after $t = 4$ days. Blow-out in a gas cavern is a rapid process: it is completed within a week or so. During such a short period of time, temperature changes are not given time enough to penetrate deep into the rock mass and, from the perspective of thermal conduction, cavern walls can be considered as a flat surface whose area equals the actual area of the cavern (More precisely, actual surface must be smoothed to eliminate shape irregularities whose radii of curvature are smaller than d), as was noted by Crotogino et al. (2001) and Krieter (2011).

- In the case of an ideal gas, $P = rT/v$ and $e = C_v T$, the heat balance equation can be written

$$m(t)C_v \dot{T}_0(t) - \dot{m}(t)rT_0 = -S_c K \int_0^t \frac{\dot{T}_0(\tau)}{\sqrt{\pi k(t-\tau)}} d\tau \quad (9)$$

where S_c is the (actual) surface of the cavern walls.

- In the case of a van der Waals gas, $P = -a/v^2 + RT/(v-b)$ and $h = C_v T - 2a/v + rT v/(v-b)$, the heat balance equation can be written

$$m(t)C_v \dot{T}_0(t) - \frac{\dot{m}(t)}{1 - bm(t)/V_0} rT_0(t) = -S_c K \int_0^t \frac{\dot{T}_0(\tau)}{\sqrt{\pi k(t-\tau)}} d\tau \quad (10)$$

¹ This equation is a basis of the SCTS software.

3. The Kanopolis blow-out

Leo Van Sambeek (2009) gave a complete account of a remarkable accident in an abandoned salt mine and provided a comprehensive and convincing explanation of that event. “On October 26, 2000, a brick factory in Kanopolis, Kansas, was substantially destroyed by bricks, sand, and water falling from the sky. The jet of air blew from a previously sealed salt mine shaft and through a pile of bricks next to the brick factory” (p. 620). Bricks and sand were blown into the air more than 100 m “for longer than 5 minutes but less than 20 minutes” (p. 621). The hypothesis analyzed by Van Sambeek was that “groundwater had entered the mine through the shafts over a long time and compressed the air within the mine” until the shaft plug collapsed in October 2000 and air escaped from the mine.

Here, we test our simplified model against the observations reported by Van Sambeek. Focus is directed to air temperature evolution in the mine and in the mine shaft. The shaft had a length of $H = 240$ m or 787 ft, and its inside dimensions were about 3.6 m \times 5.2 m or 12 ft \times 17 ft; it was lined with wood timber. An equivalent cross-section, $S = 18.72$ m², and a head-loss factor, $F = 0.225$ /m, were selected. For air, $\gamma = 1.4$. Van Sambeek suggests that compressed air (absolute) pressure might have been $P_0 = 0.272$ MPa or 39.45 psi (0.172 MPa or 24.95 psi relative) and that air volume was $V_0 = 670,000$ m³ or 4.2 MMbbls. The thickness of the air layer in the mine was 1 m. We assumed that the initial air temperature was $T_0 = 15$ °C (288 K). Equations (A4) to (A7) were used, see Appendix. Two types of computations were made. In the first series, thermodynamic evolution of air in the mine was assumed to be adiabatic (no heat exchange with the rock mass). Temperature decrease in the mine is dramatic. Figure 1 displays pressures and temperatures as a function of time during the blow-out. Cavern pressure drops from 0.27 MPa to 0.1 MPa (atmospheric pressure) in 9 minutes. The flow is not choked, and air pressure at ground level is atmospheric. Ground-level temperature drops to below -50 °C (-60 °F), a very low figure.

However, this first computation did not seem realistic enough, and heat conduction from the rock mass was taken into account in the second series of computations. Description of heat transfer raised a difficult problem. It was assumed that heat was provided by mine roof and by the dry surface of the pillars, whose overall surface is approximately, $S_c = 1,000,000$ m². No attempt was made to take into account the heat transferred from the brine that filled the lower part of the mine rooms. Results are provided on Figure 2, which displays air velocity at the casing shoe and at ground level. Here, again, the flow is normal (not choked). At ground level, the air rate decreases from 180 m/s (650 km/h) to a few m/s in 11 minutes. (This result is consistent with what was reported by Van Sambeek.) Figure 2 displays air pressure and temperature during the blow-out. In this more realistic description, which takes into account heat transfer from the mine roof, air temperature at ground level drops first to 1 °C (34 °F) before increasing to 14 °C (57 °F) at the end of the blow-out. This may explain why the air jet looked somewhat foggy (see Figure 3). Air temperature in the mine does not experience changes larger than 1 °C (1.8 °F).

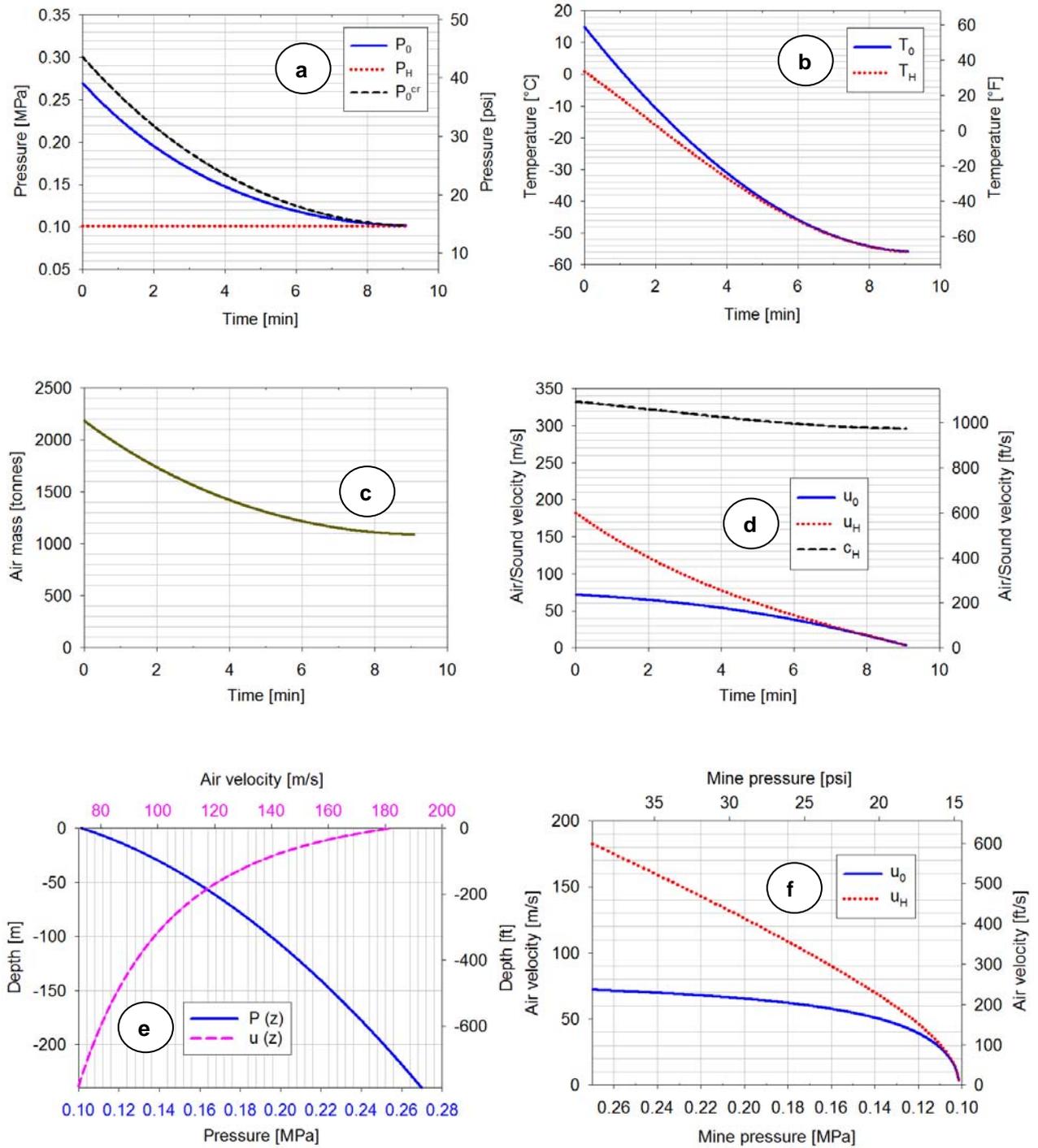


Figure 1. Kanopolis. Adiabatic assumption of air pressure (1a) - P_0^{cr} is the pressure above which the flow is choked, air temperature (1b), air mass (1c) and air velocity (1d) as a function of time, distribution of pressure and velocity as a function of depth at the starts of the blowout ($t = 0$) (1e) and distribution of air velocity as a function of air pressure in the mine (1f). Air temperature drop is unrealistic.

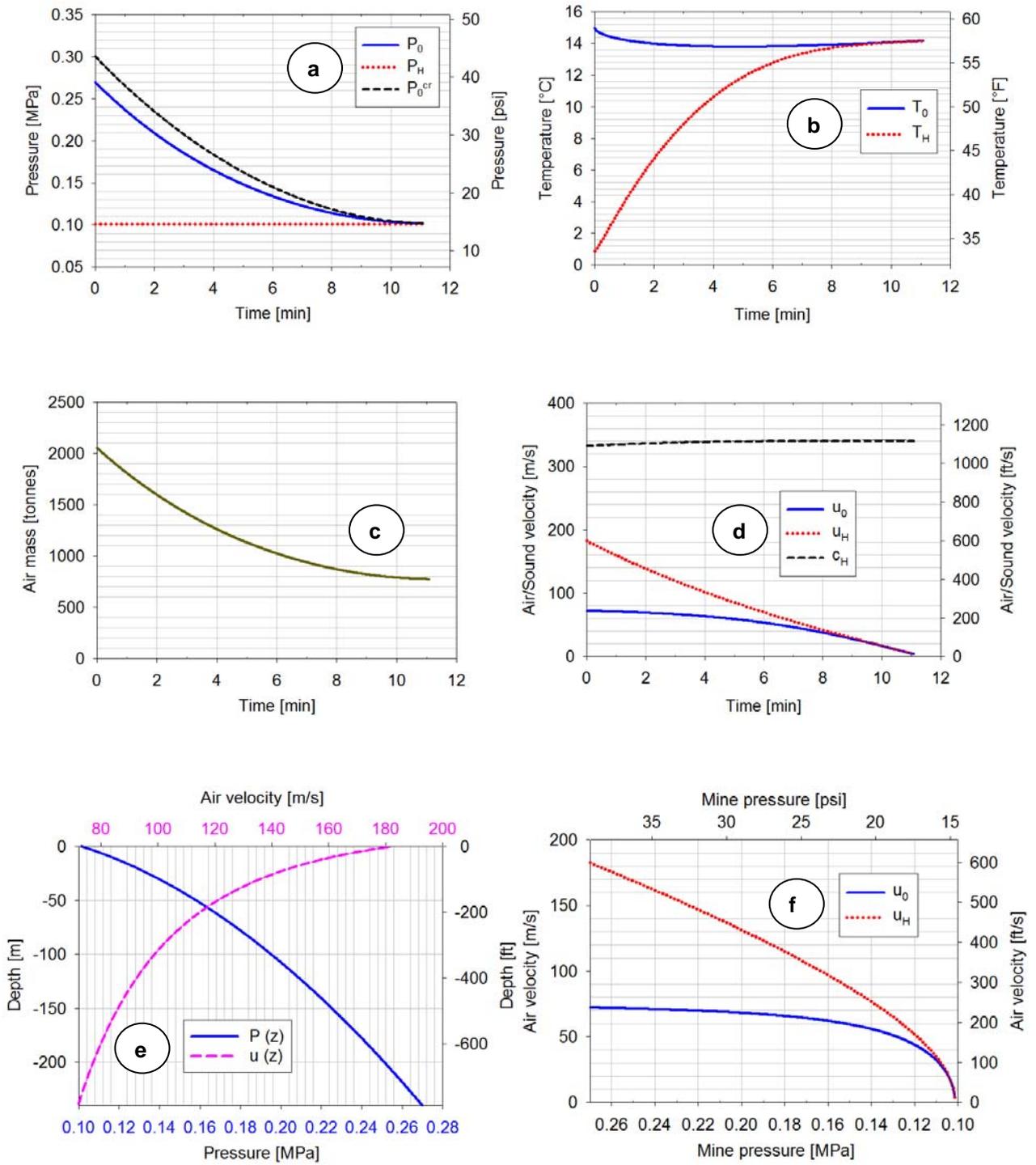


Figure 2. Kanopolis. Non adiabatic assumption – Evolution of air pressure (2a), air temperature (2b), air mass (2c) and air velocity (2d) as a function of time, distribution of pressure and velocity as a function of depth at the start of the blowout ($t = 0$) (2e) and distribution of air velocity as a function of air pressure in the mine (2f).

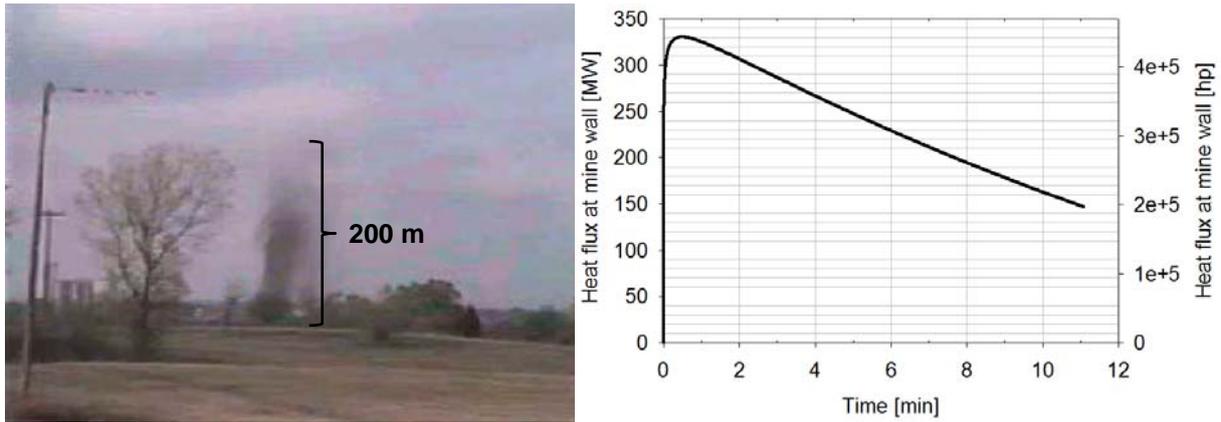


Figure 3. Air outflow during the blow-out, videotaped from a distance of 2.5 km (left, Van Sambeek, 2009) and calculated evolution of heat flux from the mine wall as a function of time (right).

4. Hydrogen blowout

A discussion of a blow-out from a generic hydrogen storage cavern is provided here. In this discussion, the cavern is cylindrical. Instead of the standard state equation of an ideal gas, a van der Waals state equation was selected, as hydrogen thermodynamic behavior exhibits some specific features of interest (in particular, an isenthalpic depressurization that leads to hydrogen warming). The parameters selected for numerical computations are presented in Table 2.

Table 2: Input parameters.

| Parameter | Symbol | Value (SI Units) | Value (US Units) |
|----------------------------|--------|--------------------------|-------------------------|
| Initial cavern pressure | P_0 | 4.5 MPa | 652 psi |
| Initial temperature cavern | T_0 | 35 °C | 95 °F |
| Cavern volume | V_0 | 1,000,000 m ³ | 6.3 MMbbls |
| Average cavern depth | H | 370 m | 1214 ft |
| Cavern overall surface | S_c | 60,000 m ² | 645,835 ft ² |
| Tubing diameter | ϕ | 7 in | |
| Friction coefficient | f | 0.01 | |

The left-hand illustration in Figure 4 shows the evolution of hydrogen specific volume as a function of time at casing-shoe depth (v_0) and at ground level (v_H). The blow-out is approximately 9-days long. The flow is choked during the first four days and is normal during the second half of the blow-out. Note that the flux of heat provided by the rock mass is especially high after 1 day (14 MW or 19000 hp).

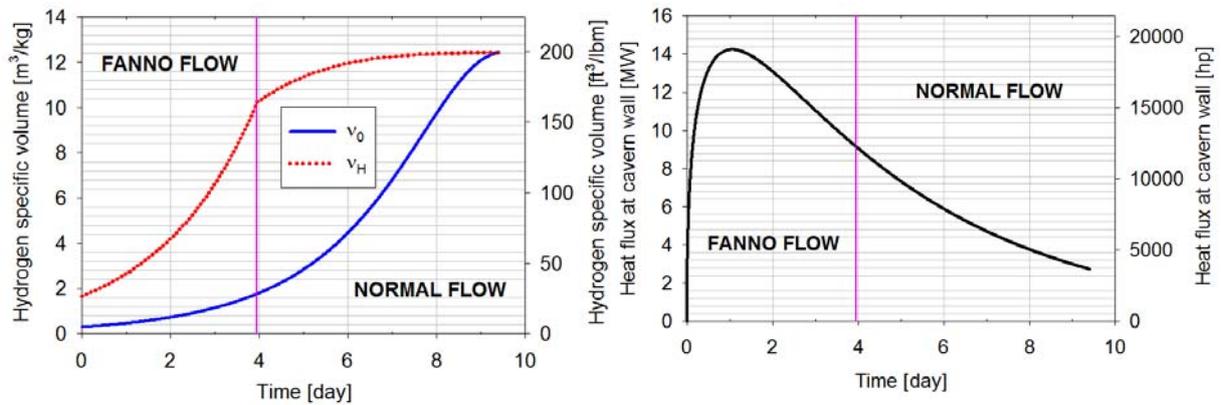


Figure 4. Evolution of specific volume as a function of time (left) and evolution of heat flux from the cavern wall as a function of time (right).

Figure 5 successively displays several parameters: hydrogen pressure in the cavern; gas depressurization in the well; hydrogen temperature; heat flux; outlet temperature; hydrogen velocity; hydrogen mass; gas pressure and specific gas volume; and cavern pressure. These are discussed below.

- Hydrogen pressure in the cavern (P_0) is 4.5 MPa when the blow-out starts and drops to 0.1 MPa (14.5 psi) after 9 days (figure 5a). Gas depressurization in the well is intense, and wellhead pressure is small even during the choked part of the blow-out. Most important is hydrogen temperature (figure 5b). In the cavern, it plummets from 35 °C (95 °F) to 20 °C (68 °F), a temperature reached after 1 day. Heat flux at this time is so large (see Figure 4) that hydrogen warms again to reach 32 °C (90 °F) at the end of the blow-out. The outlet temperature (T_H) is much colder. This is important, as, before computations were run, it was feared that hydrogen temperature in the well might increase (see Section 5).
- Hydrogen mass in the cavern decreases smoothly (figure 5c).
- Hydrogen velocity is high (figure 5d) ; it equals the celerity of sound at ground level ($u_H = c_H$), where the flow is choked. Note that sound celerity in hydrogen (more than 1200 m/s or 3900 ft/s) is much faster than in air or natural gas.
- Gas specific volume and gas pressure as a function of depth at the beginning of the blow-out are shown on figure 5e. Pressure drops down from 4.5 MPa or 653 psi (in the cavern) to 0.6 MPa or 87 psi (at ground level) when specific volume increases from 0.3 to 1.6 m³/kg (4.8 to 25.6 ft³/lbm).
- The last picture (figure 5f) displays hydrogen velocity as a function of cavern pressure during the blow-out. Note that velocities drastically drop when the flow changes from choked to normal.

The same computations were performed when assuming that hydrogen is an ideal gas (instead of a van der Waals gas). Only tiny differences were observed; however this conclusion may be wrong in some cases, as explained in the next Section.

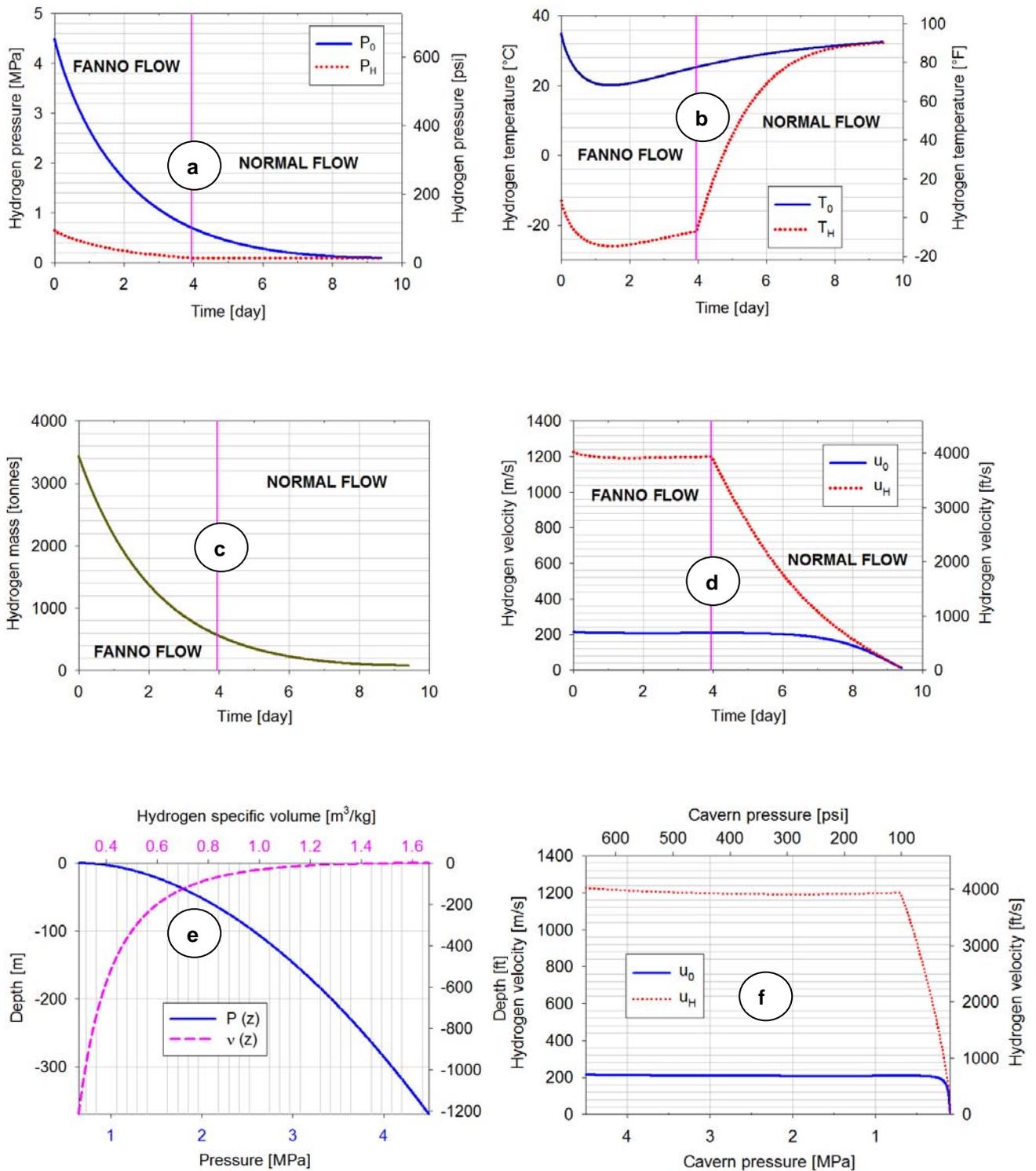


Figure 5. Evolution of hydrogen pressure (5a), hydrogen temperature (5b), hydrogen mass (5c) and hydrogen velocity (5d) as a function of time, Distribution of pressure and velocity as a function of depth at the start of the blowout ($t = 0$) (5e) and distribution of hydrogen velocity as a function of hydrogen pressure in the cavern (5f).

- **Joule-Thomson Effect**

On Figure 5b it can be observed that, except at the end of the blow-out, wellhead temperature (T_H) is much colder than cavern temperature (T_0): hydrogen temperature decreases when it travels from cavern top to ground surface. However it is known that when a real gas (as differentiated from an ideal gas) expands through a throttling device (a so-called Joule-Thomson expansion), its enthalpy remains constant and gas temperature may either decrease or increase. Gases have a Joule-Thomson inversion temperature above which gas temperature increases during an isenthalpic expansion. For hydrogen, this inversion temperature is -71°C (-96°F , much lower than the temperatures considered here).

However, the expansion of a gas during a blow-out is not a Joule-Thomson expansion, as kinetic energy cannot be neglected; in fact, the sum of the enthalpy plus the square of the gas celerity is constant, see Equation 4. It can be expected that hydrogen temperature increases during its expansion when kinetic energy can be neglected, i.e. when head losses are large.

In Figures 6a and 6b an example of a hydrogen blow-out involving gas temperature increase is presented. Parameters are the same as in Table 2, except for the friction factor which is $f = 97.3$ instead of $f = 0.01$. Gas celerity in the well is much smaller (it is divided by a factor of 10) than in the example described in Figure 5, and the cavern is emptied in one year or so (“blow-out” is somewhat of a misnomer). Hydrogen temperature is warmer at ground level than it is in the cavern, as in the case of a Joule-Thomson expansion. This solution must be considered as indicative.

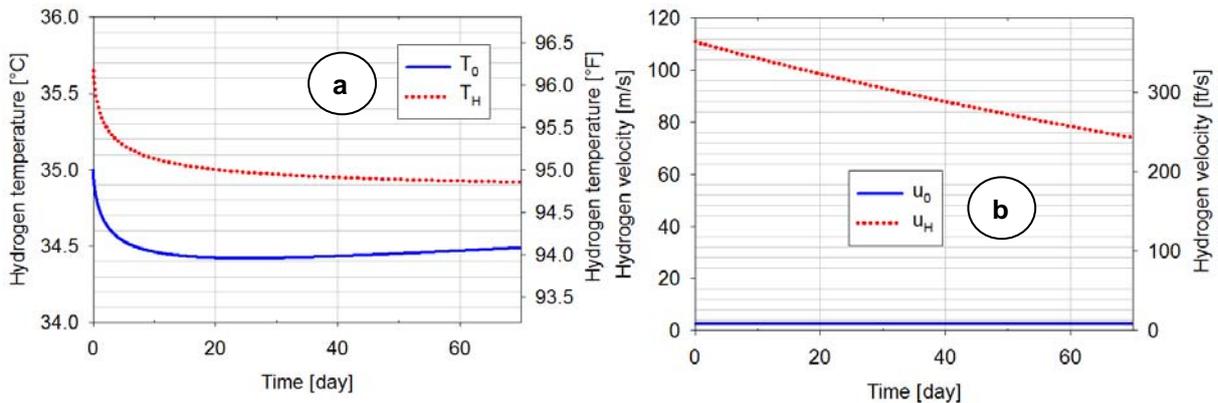


Figure 6: Evolution of hydrogen temperature (6a) and hydrogen velocity as a function of time (6b) when heat losses are large. Hydrogen is warmer at ground level than it is at cavern depth, a counter – intuitive result.

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APPENDIX

A.1. Air or natural gas flow

Taking into account the simplifications noted in Section 2.2.4, energy equation (4) can be re-written as

$$C_p T + \dot{\mu}^2 v^2 / 2 = C_p T_0 + \dot{\mu}^2 v_0^2 / 2 \quad (\text{A1})$$

or

$$P = \left(P_0 + \frac{\gamma-1}{2\gamma} \dot{\mu}^2 v_0 \right) \frac{v_0}{v} - \frac{\gamma-1}{2\gamma} \dot{\mu}^2 v \quad (\text{A2})$$

and momentum equation (5) can be written as

$$\left(\frac{P_0 v_0}{\dot{\mu}^2} + \frac{\gamma-1}{2\gamma} v_0^2 \right) \frac{1}{v^3} - \frac{\gamma+1}{2\gamma v} = F \frac{dz}{dv} \quad (\text{A3})$$

Note that this equation also can be written as: $c^2 - u^2 = \gamma \dot{\mu}^2 v^3 F \frac{dz}{dv}$. As only solutions such that $u^2 < c^2$ are considered, v is an increasing function of z .

Integration of momentum equation (A3) from $z = 0$ (casing shoe) to $z = H$ (ground level) leads to

$$\frac{1}{2} \left[\left(\frac{P_0 v_0}{\dot{\mu}^2} + \frac{\gamma-1}{2\gamma} v_0^2 \right) \left(\frac{1}{v_0^2} - \frac{1}{v_H^2} \right) \right] - \frac{\gamma+1}{4\gamma} \text{Log} \frac{v_H^2}{v_0^2} = FH \quad (\text{A4})$$

Normal flow

In addition, gas pressure at ground level is, in principle, atmospheric, $P_H = P_{atm}$:

$$P_H = \left(P_0 + \frac{\gamma-1}{2\gamma} \dot{\mu}^2 v_0 \right) \frac{v_0}{v_H} - \frac{\gamma-1}{2\gamma} \dot{\mu}^2 v_H = P_{atm} \quad (\text{A5})$$

$\dot{\mu}^2$ can be eliminated between (A4) and (A5), and v_H can be computed. However, this solution is valid only when equation (6) is true — i.e., when

$$c^2 - u^2 = \gamma P v - \dot{\mu}^2 v^2 = \left[\gamma \left(P_0 v_0 + \frac{\gamma-1}{2\gamma} \dot{\mu}^2 v_0^2 \right) - \frac{\gamma+1}{2} \dot{\mu}^2 v^2 \right] \geq 0 \quad (\text{A6})$$

Choked flow

When this condition is not met, this solution (normal flow) must be rejected. The boundary condition $P_H = P_{atm}$ cannot be satisfied anymore; instead of $P_H = P_{atm}$ or (A5), the choked-flow condition, or $C_H = u_H$, must be used:

$$c_H^2 - u_H^2 = (\gamma-1) h_0 + \frac{\gamma-1}{2} \dot{\mu}^2 v_0^2 - \frac{\gamma+1}{2} \dot{\mu}^2 v_H^2 = 0 \quad (\text{A7})$$

$\dot{\mu}^2$ can be eliminated between (A4) and (A7), and v_H can be computed. An example of this is provided on Figures A.1 and A.2. Air properties are provided in Table 1 in the main text.

On Figure A.1, cavern temperature and pressure are $T_0 = 313.15$ K and $P_0 = 0.5$ MPa, respectively, resulting in an inlet specific volume $v_0 = 0.18$ m³/kg. The well is 1000-m deep, and the friction coefficient is $F = 0.01$, resulting in $FH = 10$. Equations (A4), (A5) and (A7) allow three curves to be drawn, $\dot{\mu}^2 = \dot{\mu}^2(v_H)$. The grey zone is the “supersonic” zone in which $c_H < u_H$ (not acceptable). The intersection of the curves described by Equations (A4) and (A5), which is obtained when the exit specific volume is $v_H = 0.79$ m³/kg, can be accepted, as it is outside the supersonic zone: the flow is normal.

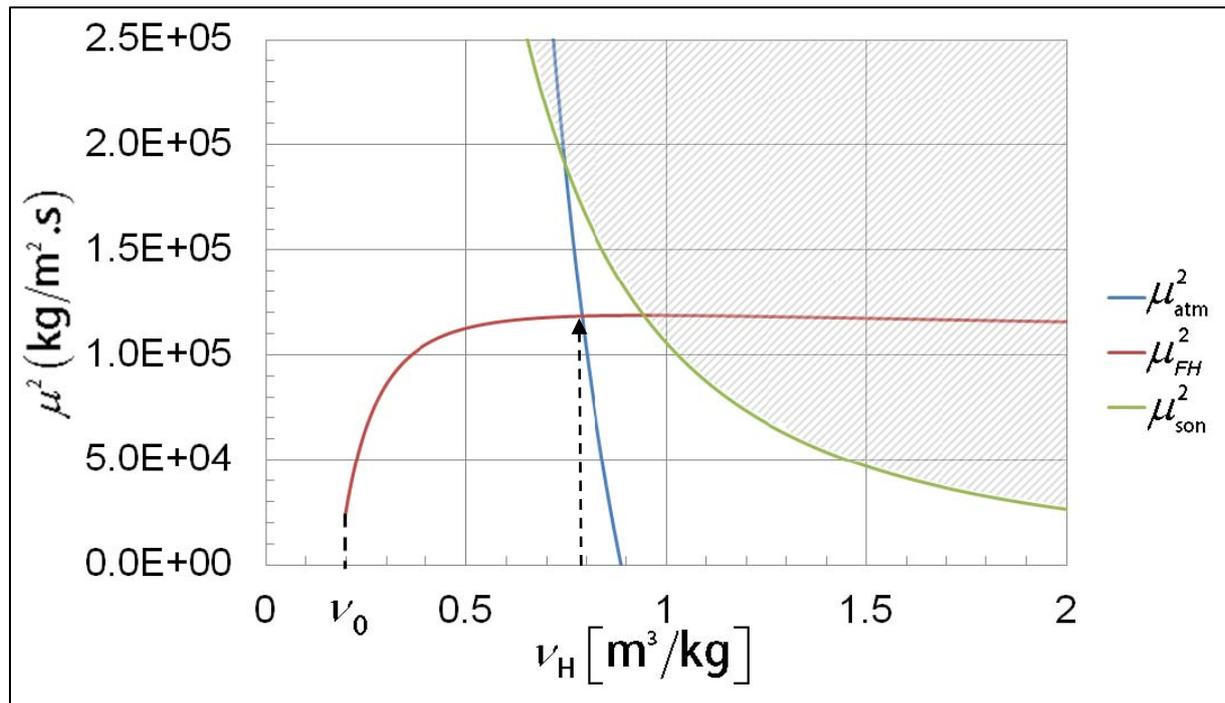


Figure A.1: Determination of the exit gas specific volume, or v_H (compressed air, normal flow).

On Figure A.2, cavern temperature and pressure are $T_0 = 313.15$ K and $P_0 = 13$ MPa, respectively, resulting in an inlet gas specific volume $v_0 = 0.007$ m³/kg. Here, again, $FH = 10$. The intersection of the curves described by (A4) and (A5) belongs to the gray supersonic zone, and the flow cannot be normal. The flow is choked, and the exit gas volume, or $v_H = 0.036$ m³/kg, is given by the intersection of the curves described by (A4) and (A7), for which the curve described by (A4) reaches a maximum.

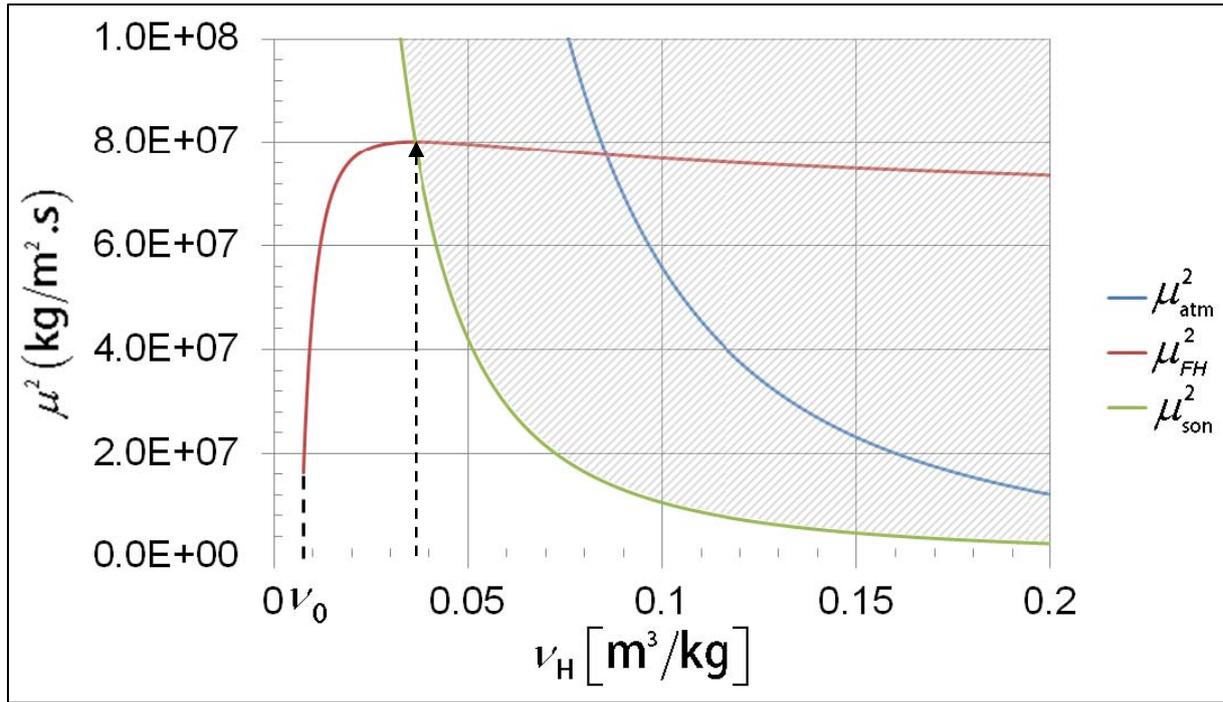


Figure A.2: Determination of the exit gas specific volume or v_H (compressed air, choked-flow).

A.2. Hydrogen flow

It is known that hydrogen behavior exhibits special characteristics: in particular, during a depressurization such that hydrogen enthalpy is constant (for instance, when hydrogen leaks from a pressurized vessel through a pinhole), hydrogen temperature *increases*. (For most gases in similar circumstances, temperature decreases). For this reason, instead of the ideal gas equation, the more precise van der Waals equation is used:

$$P = -\frac{a}{v^2} + \frac{rT}{v-b} \quad (\text{A8})$$

where a , b are two constants. It can be assumed that hydrogen temperature is above the critical temperature, which is $T_c = 33.2$ K (which means that (A8) allows to compute v when P and T are known). Hydrogen internal energy and enthalpy are

$$e(T, v) = C_v T - \frac{a}{v} \quad (\text{A9})$$

$$\text{and } h(T, v) = e + Pv = C_v T - \frac{2a}{v} + \frac{rTv}{v-b} \quad (\text{A10})$$

The same method as for air can be used: energy equation (4) allows computation of pressure and temperature. Let $\bar{C}_p = C_v + r$ (in the case of a van der Waals gas, \bar{C}_p is not the heat capacity at constant pressure or C_p) and $\bar{\gamma} = \bar{C}_p / C_v$:

$$T = (\bar{\gamma} - 1)(v - b) \frac{h_0 + \frac{1}{2} \dot{u}^2 v_0^2 + \frac{2a}{v} - \frac{1}{2} \dot{u}^2 v^2}{r(v\bar{\gamma} - b)} \quad (\text{A11})$$

$$P = -\frac{a}{v^2} + (\bar{\gamma} - 1) \frac{h_0 + \frac{1}{2} \dot{\mu}^2 v_0^2 + \frac{2a}{v} - \frac{1}{2} \dot{\mu}^2 v^2}{\bar{\gamma} v - b} \quad (\text{A12})$$

When one sets: $\psi(v, \dot{\mu}) = \frac{\bar{\gamma}}{b^2} \left(h_0 + \frac{1}{2} \dot{\mu}^2 v_0^2 + \frac{2a\bar{\gamma}}{b} \right) \left(\frac{b}{\bar{\gamma}v - b} + \text{Log} \left(\frac{v\bar{\gamma} - b}{v} \right) \right)$

It can be inferred that

$$\left[-\frac{2a}{3v^3} + (\bar{\gamma} - 1) \left[-\frac{a}{bv^2} + \psi(v, \dot{\mu}) - \frac{1}{2\bar{\gamma}} \dot{\mu}^2 \text{Log}(v\bar{\gamma} - b) - \frac{\dot{\mu}^2 v}{2(\bar{\gamma}v - b)} \right] \right] + \dot{\mu}^2 \text{Log}(v) = -F \dot{\mu}^2 \frac{dz}{dv} \quad (\text{A13})$$

and, instead of (A4), (A5) and (A6), we now have

$$\left[-\frac{2a}{3v^3} + (\bar{\gamma} - 1) \left[-\frac{a}{bv^2} + \psi(v, \dot{\mu}) - \frac{1}{2\bar{\gamma}} \dot{\mu}^2 \text{Log}(v\bar{\gamma} - b) - \frac{\dot{\mu}^2 v}{2(\bar{\gamma}v - b)} \right] + \dot{\mu}^2 \text{Log}(v) = -FH \dot{\mu}^2 \right]_{v_0}^{v_H} \quad (\text{A14})$$

$$P_H = -\frac{a}{v_H^2} + (\bar{\gamma} - 1) \frac{h_0 + \frac{1}{2} \dot{\mu}^2 v_0^2 + \frac{2a}{v_H} - \frac{1}{2} \dot{\mu}^2 v_H^2}{\bar{\gamma} v_H - b} \quad (\text{A15})$$

$$c^2 - u^2 = -\frac{2a}{v} + \frac{v^2}{(v-b)} \bar{\gamma} (\bar{\gamma} - 1) \frac{h_0 + \frac{1}{2} \dot{\mu}^2 v_0^2 + \frac{2a}{v} - \frac{1}{2} \dot{\mu}^2 v^2}{v\bar{\gamma} - b} - \dot{\mu}^2 v^2 \geq 0 \quad (\text{A16})$$

Here, again, when inequality (A16) is met (normal flow), equations (A14) and (A15) allow elimination of $\dot{\mu}^2$ and computation of v_H . When inequality (A16) is not met (choked flow), equation (A14) together with the condition $c_H^2 - u_H^2 = 0$, or

$$-\frac{2a}{v_H^3} + \frac{\bar{\gamma}(\bar{\gamma} - 1) \left(h_0 + \frac{2a}{v_H} \right)}{(v_H - b)(v_H \bar{\gamma} - b)} = \dot{\mu}^2 \left[1 + \frac{\bar{\gamma}(\bar{\gamma} - 1)(v_H^2 - v_0^2)}{2(v_H - b)(v_H \bar{\gamma} - b)} \right] \quad (\text{A17})$$

allows for computation of v_H . This is shown on Figures A.3 and A.4.

Hydrogen properties are provided in Table 1 in the main text. On Figure A.3, cavern temperature and pressure are $T_0 = 313.15$ K and $P_0 = 0.5$ MPa, respectively, resulting in an inlet hydrogen specific volume $v_0 = 2.6$ m³/kg. The well is 1000-m deep, and the friction factor is $f = 0.01$, resulting in $FH = 10$. Equations (A14), (A15) and (A17) allow three curves to be drawn, $\dot{\mu}^2 = \dot{\mu}^2(v_H)$. Here, again, the grey zone is the supersonic zone, in which $c_H < u_H$ (not acceptable). The intersection of the curves described by Equations (A14) and (A15), obtained when exit specific volume is $v_H = 11.4$ m³/kg, can be accepted, as it lays outside the supersonic zone: the flow is normal.

On Figure A.4, cavern temperature and pressure are $T_0 = 313.15$ K and $P_0 = 13$ MPa, respectively, resulting in an inlet gas specific volume $v_0 = 0.10$ m³/kg. Here, $FH = 10$. The intersection of curves (A14) and (A15) belongs to the grey supersonic zone and does not provide an acceptable solution.

The flow is choked and the exit specific volume $v_H = 0.55 \text{ m}^3/\text{kg}$, is given by the intersection of curves (A14) and (A17).

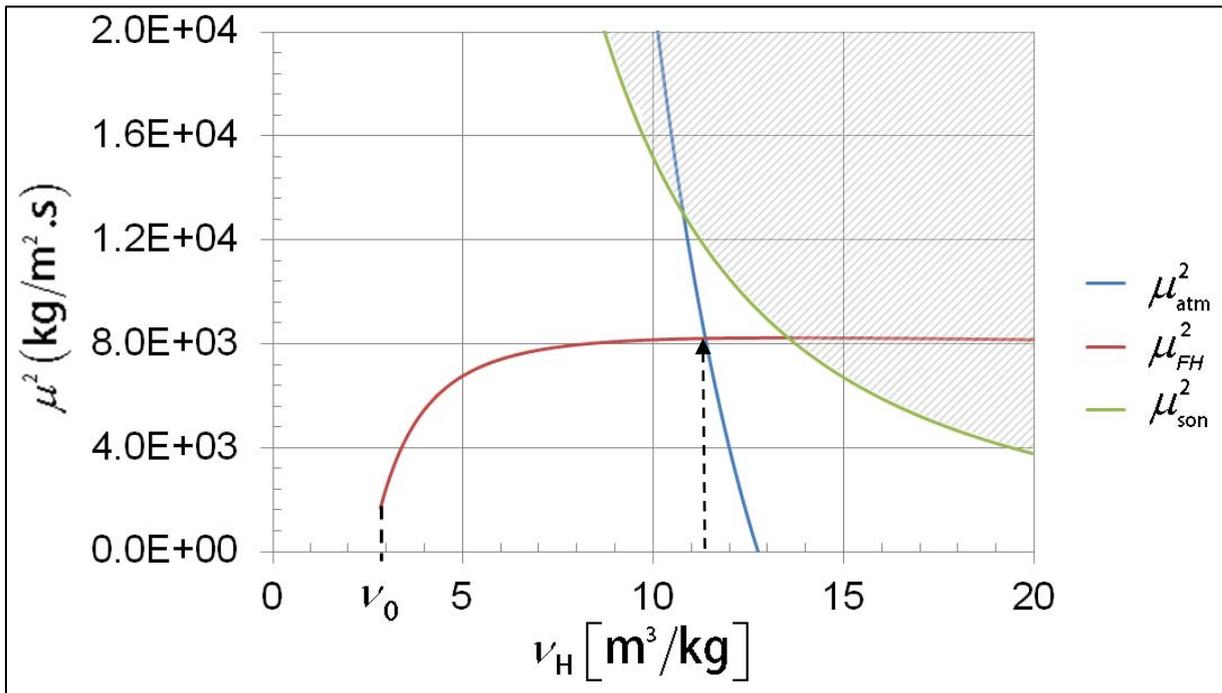


Figure A.3: Determination of the exit gas specific volume or v_H (hydrogen flow is normal).

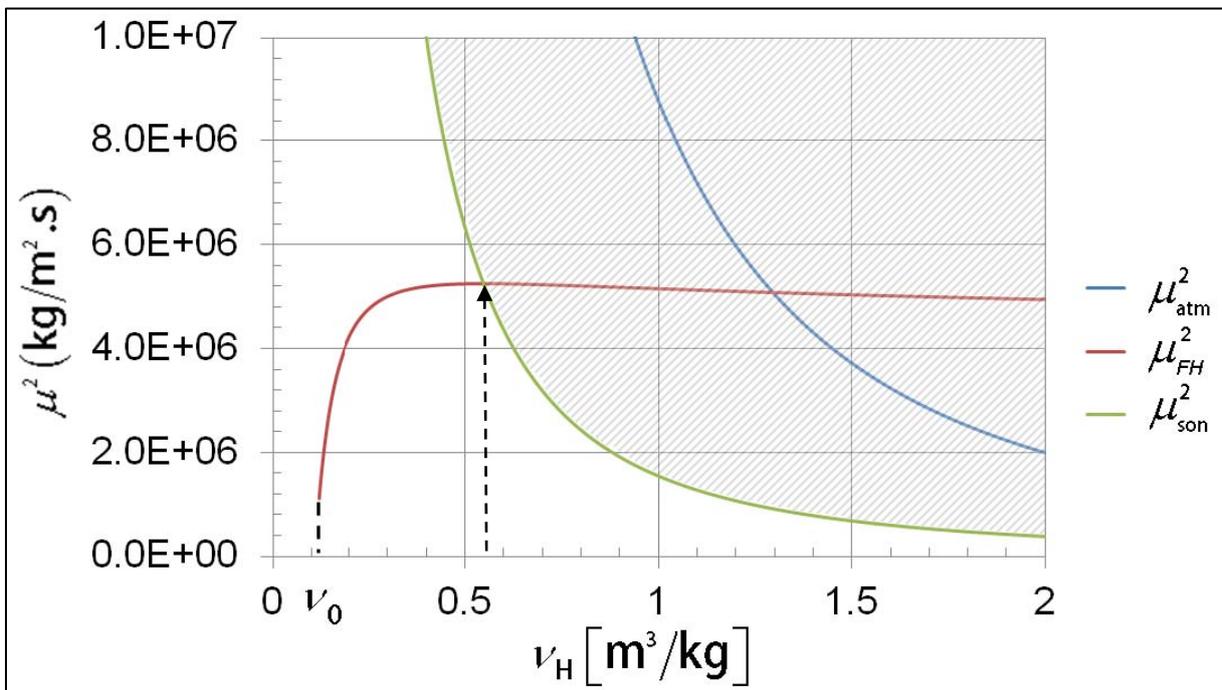


Figure A.4: Determination of the exit gas specific volume or v_H (hydrogen flow is choked).